

Review of the thesis submitted by Tetiana Korotka

in partial fulfillment of her doctoral studies,

entitled

Pole Assignment Problems in Non-Square Linear Systems

The thesis comprises 103 pages and is structured into seven chapters and two appendices. The first chapter summarizes the state-of-the-art in the field, the second one states the main goals and objectives of the thesis and the third one briefly describes the mathematical tools that are to be applied in the thesis. The original results are derived in chapters four and five; chapter six then briefly discusses the notions of controllability that are relevant to the systems under study. The seventh chapter contains a succinct statement of the original results achieved by the author. The two appendices recall further mathematical tools that are crucial in the comprehension of the work. The bibliography section lists 4 references to the work of the author as well as 44 references to the work of other authors.

Thus, the thesis is well and logically structured. This is important in view of the complexity of the problem studied and the deep knowledge of linear systems theory required. The problem of pole assignment in linear systems using proportional state feedback is a fundamental and prototype problem to which many other problems of control can be reduced. No doubt the problem was given a due attention in the literature. Studied was not only a simpler problem of assigning the pole positions but also the more difficult problem of assigning the structure of the poles, that is, the position as well as the multiplicity of the individual poles. The latter problem describes in fact the limits of the proportional state feedback in altering the dynamics of linear systems.

The thesis deals with the general case of linear systems that are governed by an implicit system of state-space differential equations in which the number of equations does not equal the number of state variables. Such systems are frequently encountered in economics, robotics, and other areas of applications. While the difficulties caused by the implicit nature of the system have been overcome during the last decades, the difficulty of pole placement arising from the over-determined or under-determined systems of equations is subject to current research.

The formulation of the pole assignment problem in such general non-square linear systems first calls for an appropriate modification of some notions that are used to study square linear systems. To this effect, the notions of regularity and controllability are generalized to non-square systems, including the reasons for such a generalization. This step then makes it

possible to generalize the notion of the characteristic polynomial so that one can speak of the poles of the system.

The basic mathematical tool to solve the problem is a specific standard form of the system, referred to as the feedback canonical form, to which any system can be brought by means of a change of basis in the space of states, inputs and outputs. In the case of non-square systems, this form defines six sets of feedback invariants, which disappear under specific assumptions and, in particular, reduce to a single set of feedback invariants, called the controllability indexes, in the case of square explicit systems.

The original results of the thesis are formulated in Theorems 3, 4, 5 and 11. Theorem 3 states necessary conditions under which a proportional state feedback exists that assigns the desired pole positions. Sufficient conditions for several particular cases are provided in Theorem 4. Theorem 5, in turn, gives a necessary and sufficient condition to solve the pole assignment problem in the special case of real fixed invariant poles of the system. It is to be noted that the proofs of sufficiency are constructive and hint a method to calculate a state feedback that achieves the desired pole positions. Finally, Theorem 11 deals with the more difficult problem of pole structure assignment and provides sufficient conditions to solve the problem under several special assumptions.

The results achieved are correct, rigorously proven, and advance the state-of-the-art of the pole/structure assignment problem in linear systems by means of proportional state feedback. Although not definitive, these results are valuable and are respected by the community. This is demonstrated by the publications of the author to appear in an impacted journal and in the proceedings volume of a prestigious conference.

The thesis is submitted in English and needs very little editing. On the formal side, there are few inconsistencies only. For example, Theorem 1 is subsequently referred to as Proposition 1 and the proof of Theorem 2 wrongly refers to equation (5.16).

The thesis fully demonstrates the abilities of the author to carry out research and to achieve original results. Therefore, I am pleased to recommend the Faculty of Mechatronics, Informatics and Interdisciplinary Studies, Technical University of Liberec to confer on Tetiana Korotka the degree of Doctor (Ph.D.) in the field of Engineering Cybernetics.

Prague, November 26, 2012



Prof. Ing. Vladimír Kučera, DrSc., dr.h.c.

Opponent review of a doctoral thesis

Theme of the thesis: Pole assignment problems
in non-square linear systems

Author of the thesis: Ing. Tetiana Korotka

Supervisor: Ing. Petr Zagalak CSc.

Opponent: Ing. Jan Ježek CSc.

The thesis is devoted to the methods of changing the dynamic behavior of the linear generalized state-space systems, namely to pole assignment by state feedback. For the "classical" state-space systems, described by equation

$$\dot{x} = Ax + Bu,$$

the problem is well-known and solved, however, for generalized systems

$$E\dot{x} = Ax + Bu$$

(matrices E, A possibly non-square), the situation is not so clear and problem are not yet completely solved.

The generalized systems, according to degree of generalization, exhibit new properties, not present in the classical systems:

- the solution (the state trajectories - function of time) may contain not only usual "slow" functions (say, piecewise continuously differentiable) but also a "fast" part - distributional functions (Dirac impulses and their derivatives)
- the system may contain also a non-dynamical part
- given the initial state and the input signal, the solution may be non-unique, here is "an internal degree of freedom"

- for some initial states and inputs, the solution may not exist at all. In such a case, the equations of the system are in contradiction.

For the pole assignment problems, the concepts of controllability and of controllability indices play a key role. This was introduced by Rosenbrock and elaborated by others, including Kučera, Zagalak and Loiseau. For the generalized systems, even the definition of controllability causes problems and varies in various authors. There is more values than the indices playing a role here.

The theme of the thesis is very actual. The solution contributes both to the development of science, and for applications, e.g. in engineering, economy, biology and physics. The thesis contains some new results - Theorem 3 and 5. It can be seen that the author mastered the complicated mathematics and used it to obtain new results. I appreciate also the mathematical exactness throughout the work. The explanations are clear, the illustrative examples are well chosen. The English language is very good.

Conclusion:

I recommend the thesis to be defended.

Review comments:

- 1) Why the assumption of real roots in Theorem 5?
I agree that the complex roots may cause some complications but I think no difficult ones.
- 2) As an opponent, I am known by usually including a long list of errors and suggestions for improving the readability. In this case, the list is very short and contains mainly the latters.

<u>Page</u>	<u>It is:</u>	<u>It should be better:</u>
1	(E, A, B) is called explicit (E, A, B) is called regular (E, A, B) is called regularizable	square (E, A, B) is called --- square (E, A, B) is called --- square (E, A, B) is called ---
4	the Smith-McMillan for	should be included in the mathematical preliminaries
10 and similarly elsewhere	$(b_i) \begin{bmatrix} & & \\ & & \\ & & s - a_{ij} b_j \end{bmatrix}$ $\alpha_i(s) = s^{l_i} + a_{i, l_i-1} s^{l_i-1}$ <p>the running index is somewhere i but elsewhere j</p>	$\begin{bmatrix} & & \\ & & \\ & & s - a_{ij} b_j - 1 \end{bmatrix}$ $\dots + a_{i, l_i-1} s^{l_i-1}$ <p>to unify</p>
21	Remark 2. Proposition 1	Theorem 1
55		regularizability
69	Accordingly --- ... the row and column regularizable ---	weakly regularizable
90		regulariz- able
92	matrix A(s) is irreducible	should be introduced above
92	Definition 25. Such a matrix	Matrix $M(s) \in \mathbb{R}^{n \times m}(s)$
	Definition 28 IF	IF should be included in the list of notations, page <u>VII</u>

93 The matrix A_c above
is in the rational form

2

95

solvability

96

Transformations P, Q
(PEQ, PAQ, PB)

P, Q, G
($---- PBG$)

Prague, 4/12/2012

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